ISSN 2229-5518

Relay Based Identification of Systems

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Abstract - In this paper, describing functions (DF) of relay with hysteresis is used for identification of systems. Describing function analysis is widely adopted in relay feedback-based identification methods because of the ease of computation involved and the general usefulness of the method. In process control systems, the noises come from measuring devices, control valves or the process itself. During the relay feedback experiments, amplitude of the limit cycle output is often corrupted with noises which may even fail the test. To overcome the possible failure, a relay with hysteresis is considered in the proposed identification method. Both off-line and on-line identification methods are presented to show the advantages of the online identification. The identification method is developed for a first order plus time delay (FOPDT) system although the method can be extended for higher order systems in a straightforward manner. A Simulation example is included to illustrate the use of the proposed method for both off-line and on-line identification.

Index Terms – Describing function, FOPDT, Hysteresis, Identification, Off-line, On-line, Relay.

1 INTRODUCTION

THE normal use of relay feedback is to induce a limit cycle in process output. The limit cycle is an important behavior which is used to extract process information. From the measurements made on this limit cycle, process models are obtained. For analysis, the nonlinear device relay is approximated by an equivalent gain using the describing function (DF) technique. The describing function technique is used by several authors to identify the transfer function models of stable and unstable processes. A lower order transfer function model of the process dynamics can conveniently be obtained from relay feedback experiment. In early eighties, Astrom and Hagglund [1] proposed the use of relay feedback combined with a describing function approximation as a simple means to determine the ultimate gain and ultimate frequency. Luyben [2] pioneered the use of relay feedback tests and describing function analysis for system identification. A number of relay based identification methods have been proposed to obtain process models in terms of transfer functions. Several authors have used the approximate describing function method with or without involving modified relays for estimating process transfer function models. Two relay tests are used by Li et al [3] to identify an equivalent model of stable and unstable process. However the use of an additional relay test is tedious and time consuming. Although, the relay auto-tuning is improved by some authors using modified relay based identification or by using Fourier analysis or by exact analysis, the improvements do not overcome the practical constraints of the conventional relay based identification. The conventional relay method is an off-line identification method.Off-line tuning affects the operational process regulation which may not be acceptable for certain critical applications. Therefore the tuning under tight continuous closed loop control that is on-line is necessary. Measurement noise is an important issue in identification problem. Accurate measurement of the process frequency response can be done if smooth limit cycle is produced. One way to reduce the effect of measurement noise is by using relay with hysteresis [1]. To improve accuracy of the identification methods, some exact methods such as time-domain approach [5], exact waveform method [6], z-transform method [4], state space method [7] and using the effect of shape factor [8, 9] are proposed. However, unlike the DF method, the exact methods are not straightforward and are time consuming. It is extremely difficult to obtain simple explicit expressions for process model parameters by exact analysis methods. In this paper, a new method of identification by using relay with hysteresis is proposed for both offline and online tuning by using describing function analysis.

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2IDENTIFICATION METHODS

This section presents the procedures for identification of a FOPDT model by the off-line and on-line relay based closed loop tests.

2.1Off-line identification method

Let the dynamics of G(s) in the conventional off-line identification structure shown in Fig. 1 be represented by the FOPDT process model transfer function

$$G_m(s)=\frac{Ke^{-Ds}}{Ts+1} \quad (1)$$

where K, T and D are the steady state gain, time constant and the time delay, respectively.

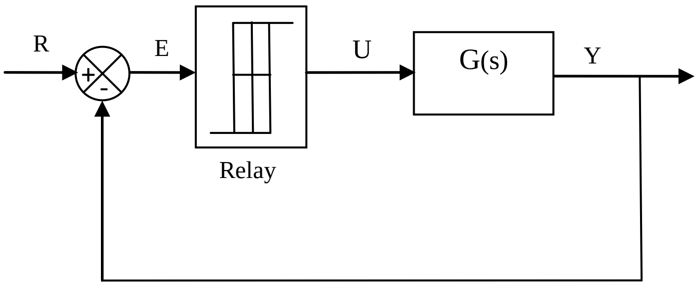

Fig. 1. Conventional Off-line identification structure

Estimation of T and D

From the measurements of peak amplitude (A) and time period (Pu) of the limit cycle output signal, T and D of the process models can be estimated for any non-zero settings of the relay height and hysteresis i.e. h ≠ 0 and ε≠ 0. In order for a periodic solution to correspond to a stable limit cycle

$NG_m(j\omega) = -1$ (2)

where N is the equivalent gain of a relay with hysteresis obtained by the DF as

$$N = \frac{4h(\sqrt{A^2-\varepsilon^2}-j\varepsilon)}{\pi A^2} \quad (3)$$

Substitution of Gm (jω) and N in (2) results in

$$\frac{4h(\sqrt{A^2-\varepsilon^2}-j\varepsilon)}{\pi A^2}\frac{Ke^{-j\omega D}}{j\omega T+1} = -1 \quad (4)$$

Equating the magnitude and phase angle of both sides of (4), one obtains

$$T = \frac{\sqrt{\left(\frac{4hK}{\pi A}\right)^2-1}}{\omega} \quad (5)$$

$$D = \frac{\pi-\tan^{-1}(\omega T)-\tan^{-1}\left(\frac{\varepsilon}{\sqrt{A^2-\varepsilon^2}}\right)}{\omega} \quad (6)$$

where h is relay height, ε is the hysteresis and

$$\omega = \frac{2\pi}{P_u}$$

2.2 On-line identification method

As shown in Fig.2, the on-line identification structure has a PID controller connected in parallel with the relay in the loop. Let the form of the PID controller be

$$G_c(s) = K_c\left(1+\frac{1}{T_I s}+\frac{T_d s}{\alpha T_d s+1}\right) \quad (7)$$

where K_C, T_I, T_d and α are proportional gain, integral time constant, derivative time constant and derivative filter constant,respectively. Normally, α is very small, so the derivative filter term in (7) is neglected in the following for ease in analysis. Fig. 3 shows equivalent representation of Fig. 2. It is apparent from Fig. 3 that the relay sees a fictitious process $\overline{G}(s)$ (G(s) coupled with the inner loop controller Gc (s)) in the loop. The process gets stabilizing signal from the inner feedback controller Gc (s) thereby improving its stability during identification.

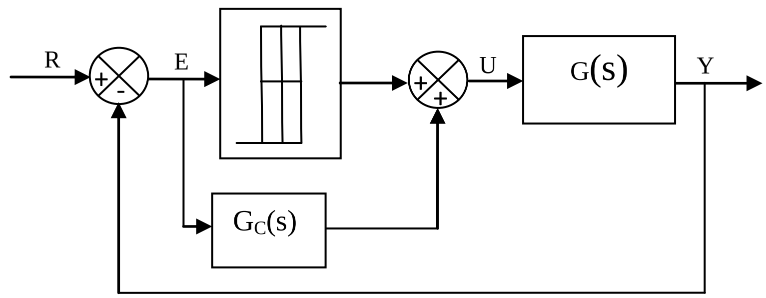

Fig. 2. On-line identification structure

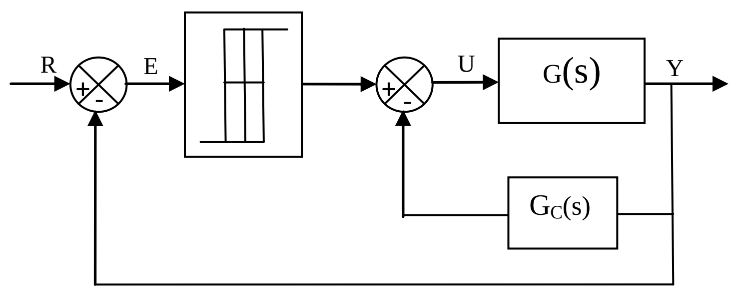

Fig. 3. Equivalent representation of Fig. 2

Estimation of T and D

Again, in order for a periodic solution to correspond to a stable limit cycle

$N\bar{G}_m(j\omega) = -1$ (8)

where

$$\bar{G}_m(j\omega) = \frac{G_m(j\omega)}{1+G_c(j\omega)G_m(j\omega)} \quad (9)$$

It is to be noted that the oscillation frequency ω varies with different controller settings. Then, (8) becomes

$$G_m(j\omega)[N+G_c(j\omega)] = -1 \quad (10)$$

Substitution of Gm (jω), N and Gc (jω) in (10) and solving gives

$$\frac{Ke^{-j\omega D}}{j\omega T+1}(a+jb)=-1 \quad (11)$$

where

$$a=\frac{4h\sqrt{A^2-\varepsilon^2}}{\pi A^2}+K_c$$

$$b=K_c\omega T_d-\frac{K_c}{\omega T_I}-\frac{4h\varepsilon}{\pi A^2}$$

Equating the magnitude and phase angle of both sides of (11), one obtains

$$T=\frac{\sqrt{K^2(a^2+b^2)-1}}{\omega} \quad (12)$$

$$D=\frac{\pi+\tan^{-1}\left(\frac{b}{a}\right)-\tan^{-1}(\omega T)}{\omega} \quad (13)$$

The steady state gain K of the process is assumed to be known a priori or obtained by some other method. The remaining parameters (T, D) of the process model are estimated using (5) and (6) for the off-line identification method and (12) and (13) for the on-line identification method.

Initial PID controller parameters

The relay induces stable limit cycle output when the fictitious process model $\bar{G}_m(s)$satisfies (8). Using (1) and (7) and setting Td = 1 and TI = 1, $\bar{G}_m(s)$ can be expressed as

$$\bar{G}_m(s)=\frac{sKe^{-Ds}}{s(Ts+1)+KK_c(s^2+s+1)e^{-Ds}} \quad (14)$$

For a small value of Kc, (14) becomes

$$\bar{G}_m(s)=\frac{Ke^{-Ds}}{Ts+1} \quad (15)$$

It is evident from (15) that a positive value of $\frac{D}{T}$ ensures a limit cycle condition for stable processes. From simulation studies, it has been found that the choice of controller parameters Kc= 0.1, TI = 1 and Td = 1, results in a stable limit cycle output.

3SIMULATION RESULTS

Consider a typical second order process dynamics [3]$G(s)=\frac{e^{-2s}}{(10s+1)(s+1)}$ for identification. The FOPDT model parameters obtained by Li et al.'s method are K = -0.501, T = -5.03 and D = 2. Since the model is not valid, they have recommended a second order plus time delay model for the process dynamics by following two relay tests. Liu et al. [10] also proposed a second order plus time delay model.Employing the relay with parameters (h, ε) = (1, 0.1), the relay test is carried out and the limit cycle data are measured from Fig. 4 for off-line identification and from Fig. 5 for on-line identification.It is assumed that K is known a priori. Using (5-6) and (12-13) the FOPDT model parameters are estimated and tabulated in Table 1. This clearly indicates that the proposed method can successfully be used to identify simple transfer function model.Nyquist curves of the actual process and the identified modelsare shown in Fig.6.

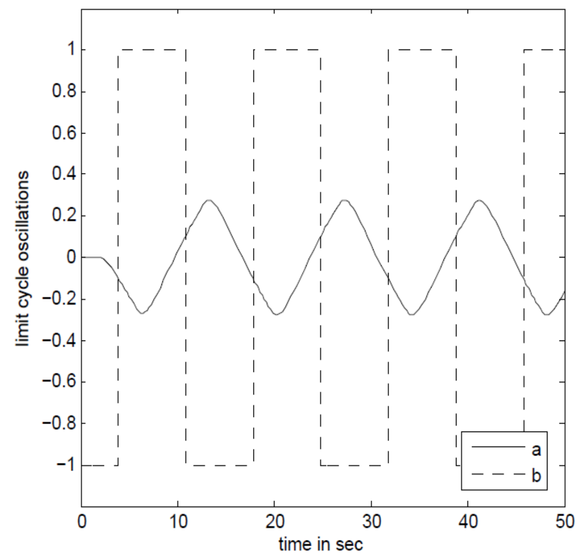

Fig. 4. Typical limit cycle waveforms for off-line identification, (a) y (t) and (b) u (t) for given example

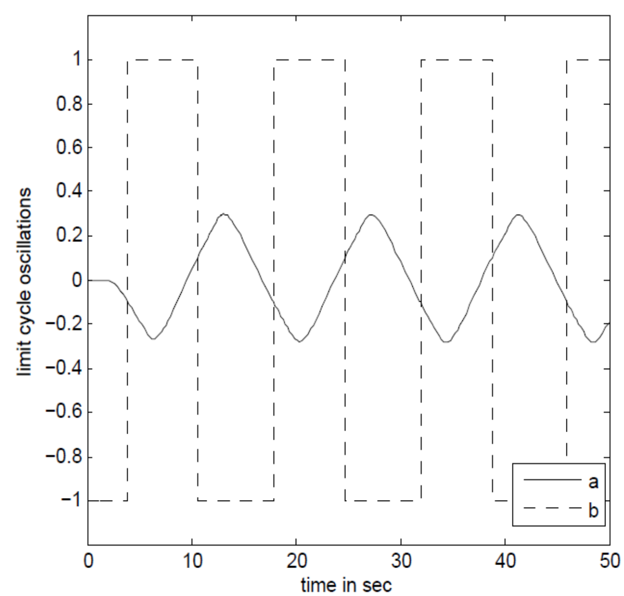

Fig. 5. Typical limit cycle waveforms for on-line identification, (a) y (t) and (b) u (t) for given example

TABLE. 1

PROCESS MODEL PARAMETERS

Identification Methods	Parameters
Off-line Identification	K = 1, T = 10.24, D = 3.12
On-line Identification	K = 1, T = 9.81, D = 3.15

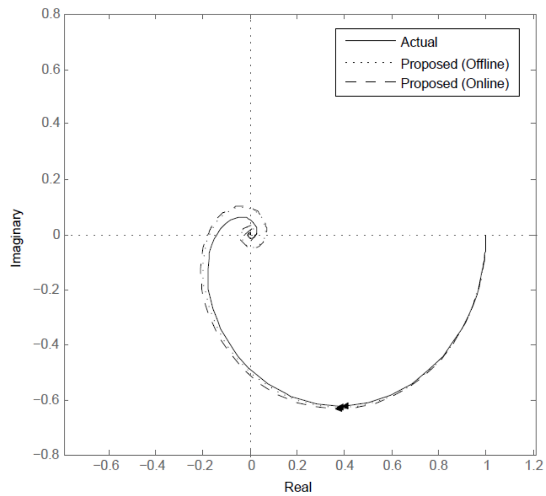

Fig. 6. Nyquist curves of process models and the process

4CONCLUSION

In this paper,a relay with hysteresis is used for identification of systems using describing function (DF) technique. Both off-line and on-line identification methods are used separately to identify the parameters of a second order process in terms of a first order process transfer function model. Further, based on the identified models the PID controller parameters can be updated on demand when the PID tuning rules are available in terms of the model parameters.

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